

## What underlies successful word problem solving? A path analysis in sixth grade students



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### ARTICLE INFO

#### Article history:

Available online 26 May 2013

#### Keywords:

Word problem solving  
Visual-schematic representations  
Relational processing  
Spatial ability  
Reading comprehension

### ABSTRACT

Two component skills are thought to be necessary for successful word problem solving: (1) the production of visual-schematic representations and (2) the derivation of the correct relations between the solution-relevant elements from the text base. The first component skill is grounded in the visual-spatial domain, and presumed to be influenced by spatial ability, whereas the latter is seated in the linguistic-semantic domain, and presumed to be influenced by reading comprehension. These component skills as well as their underlying basic abilities are examined in 128 sixth grade students through path analysis. The results of the path analysis showed that both component skills and their underlying basic abilities explained 49% of students' word problem solving performance. Furthermore, spatial ability and reading comprehension both had a direct and an indirect relation (via the component skills) with word problem solving performance. These results contribute to the development of instruction methods that help students using these components while solving word problems.

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### 1. Mathematical word problem solving

Mathematical word problem solving plays a prominent role in contemporary mathematics education (Rasmussen & King, 2000; Timmermans, Van Lieshout, & Verhoeven, 2007). The term *word problem* is used to refer to any math exercise where significant background information on the problem is presented as text rather than in mathematical notation. As word problems often involve a narrative of some sort, they are occasionally also referred to as *story problems* (Verschaffel, Greer, & De Corte, 2000). An example of a word problem is given below (taken from Hegarty & Kozhevnikov, 1999):

**Example 1.** At each of the two ends of a straight path, a man planted a tree and then, every 5 m along the path, he planted another tree. The length of the path is 15 m. How many trees were planted?

Students often experience difficulties in the understanding of the text of a word problem, rather than its solution (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1981; Lewis & Mayer, 1987). Two component skills are thought to be necessary for successful word problem solving: (1) producing visual-schematic representations (e.g., Hegarty & Kozhevnikov, 1999; Krawec, 2010; Montague

& Applegate, 2000; Van Garderen & Montague, 2003) and (2) relational processing, that is deriving the correct relations between the solution-relevant elements from the text base (e.g., Hegarty, Mayer, & Monk, 1995; Kintsch, 1998; Van der Schoot, Bakker Arkema, Horsley, & Van Lieshout, 2009; Verschaffel, 1994; Verschaffel, De Corte, & Pauwels, 1992). These two component skills are presumed to explain unique variance in students' word problem solving performance and cover different processing domains (Hegarty & Kozhevnikov, 1999; Krawec, 2010; Van der Schoot et al., 2009). The production of visual-schematic representations is grounded in the visual-spatial domain (e.g., Hegarty & Kozhevnikov, 1999; Krawec, 2010; Mayer, 1985; Van Garderen, 2006), whereas relational processing is seated in the linguistic-semantic domain (e.g., Pape, 2003; Thevenot, 2010; Van der Schoot et al., 2009). These component skills, as well as the basic abilities which underlie each of these skills, are described below.

#### 1.1. Component skill in the visuo-spatial domain: The production of visual-schematic representations

Rather than the superficial selection of numbers and relational keywords from the word problem text (often resulting in the execution of the wrong arithmetic operations), good word problem solvers generally construct a visual representation of the problem to facilitate understanding (e.g., Hegarty & Kozhevnikov, 1999; Krawec, 2010; Montague & Applegate, 2000; Van der Schoot et al., 2009). With this, the nature of these visual representations determines their effectiveness. According to Hegarty and

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Kozhevnikov (1999), two types of visual representations exist: *pictorial* and *visual-schematic* representations. Children who create pictorial representations tend to focus on the visual appearance of the given elements in the word problem. These representations consist of vivid and detailed visual images (Hegarty & Kozhevnikov, 1999; Presmeg, 1997). However, several studies have reported that the production of pictorial representations is negatively related to word problem solving performance (Ahmad, Tarmizi, & Nawawi, 2010; Hegarty & Kozhevnikov, 1999; Kozhevnikov, Hegarty, & Mayer, 2002; Krawec, 2010; Van Garderen, 2006; Van Garderen & Montague, 2003). An explanation for this finding is that children who make pictorial representations fail to form a coherent visualization of the described problem situation and base their representations solely on a specific element or sentence in the word problem text (Hegarty & Kozhevnikov, 1999; Krawec, 2010; Van Garderen, 2006; Van Garderen & Montague, 2003). Children who make visual-schematic representations do integrate the solution-relevant text elements into a coherent visualization of the word problem (e.g., Ahmad et al., 2010; Krawec, 2010; Van Garderen, 2006). This explains why, in contrast to the production of pictorial representations, the production of visual-schematic representations is found to be positively related to word problem solving performance (Hegarty & Kozhevnikov, 1999; Van Garderen, 2006; Van Garderen & Montague, 2003).

#### 1.1.1. Basic ability in the visuo-spatial domain: Spatial abilities

The production of visual-schematic representations is influenced by spatial ability. Children with good spatial skills have been found to be better able to make visual-schematic representations than children with poor spatial skills (e.g., Hegarty & Kozhevnikov, 1999; Krawec, 2010; Van Garderen, 2006; Van Garderen & Montague, 2003). Although there are many definitions of what spatial ability is, it is generally accepted to be related to skills involving the retrieval, retention and transformation of visual information in a spatial context (Velez, Silver, & Tremaine, 2005). Especially the involvement of a specific spatial factor – that is, *spatial visualization* – in making coherent visual-schematic representations has been made clear by several authors (Hegarty & Kozhevnikov, 1999; Krawec, 2010; Van Garderen, 2006; Van Garderen & Montague, 2003). Spatial visualization refers to the ability to mentally manipulate objects (i.e. mental rotation; Kaufmann, 2007; Voyer, Voyer, & Bryden, 1995). In the present study, spatial ability refers to spatial visualization as described above.

Besides the role of spatial ability in word problem solving via the production of visual-schematic representations, several authors also report a *direct* relation between spatial ability and word problem solving (Battista, 1990; Blatto-Vallee, Kelly, Gaustad, Porter, & Fonzi, 2007; Booth & Thomas, 1999; Edens & Potter, 2008; Geary, Sauls, Liu, & Hoard, 2000; Hegarty & Kozhevnikov, 1999; Orde, 1997). Blatto-Vallee et al. (2007), for example, showed that spatial abilities explained almost 20% of unique variance in word problem solving performance. Casey and colleagues revealed that the direct role of spatial abilities in word problem solving lies in performing the actual mathematical operations and numerical reasoning (e.g., Casey, Nuttall, & Pezaris, 1997, 2001; Casey et al., 2008).

#### 1.2. Component skill in the linguistic-semantic domain: Relational processing

Although the production of visual-schematic representations is a necessary condition for successful word problem solving, it is not always a sufficient condition (Kintsch, 1998; Pape, 2003; Van der Schoot et al., 2009), since children may be very well capable of forming a *visual-schematic* representation without being able to infer the *correct* relations between the solution-relevant elements

from the word problem text (Coquin-Viennot & Moreau, 2003; Krawec, 2010; Thevenot, 2010). Relational processing in word problem solving can be effectively revealed in word problems in which the relational term maps onto non-obvious mathematical operations (De Corte, Verschaffel, & De Win, 1985; Kintsch, 1998; Thevenot, 2010; Thevenot & Oakhill, 2005, 2006; Van der Schoot et al., 2009). In word problems with an obvious mapping, it is sufficient to first select the numbers and relational terms from the text and then to directly translate these into a set of computations (Hegarty et al., 1995; Pape, 2003; Van der Schoot et al., 2009). However, in non-obvious word problems, other text elements are necessary for the construction of an effective mental model of the word problem including the appropriate relations between the key variables (De Corte et al., 1985; Thevenot, 2010; Thevenot & Oakhill, 2005, 2006; Van der Schoot et al., 2009). Consider, for example, the following word problem in which the relation term ‘more than’ primes an inappropriate mathematical operation:

**Example 2.** At the grocery store, a bottle of olive oil costs 7 euro.

That is 2 euro ‘more than’ at the supermarket.

If you need to buy 7 bottles of olive oil, how much will it cost at the supermarket?

In this so-called inconsistent word problem (Hegarty, Mayer, & Green, 1992; Hegarty et al., 1995; Kintsch, 1998; Van der Schoot et al., 2009), the crucial arithmetic operation (i.e. 7-2) cannot be simply derived from the relational keyword (‘more than’). Rather than making use of a superficial, direct-retrieval strategy (Giroux & Ste-Marie, 2001; Hegarty et al., 1995; Thevenot, 2010; Verschaffel, 1994; Verschaffel et al., 1992), problem solvers have to appeal to a problem-model strategy in which they translate the problem statement into a qualitative mental model of the base type of situation (in this case: a subtraction situation) that is hidden in the problem. Here, this translation requires the identification of the pronominal reference ‘that is’ as the indicator of the relation between the value of the first variable (‘the price of a bottle of olive oil at the grocery store’) and the second (‘the price of a bottle of olive oil at the supermarket’). On the basis of the constructed mental model, problem solvers are then able to plan and execute the required arithmetic operations. Hence, inconsistent word problems are suitable to measure relational processing.

#### 1.2.1. Basic ability in the linguistic-semantic domain: Reading comprehension

Previous studies have shown that the role of relational processing in word problem solving is influenced by a child’s reading comprehension abilities (e.g., Lee, Ng, & Lim, 2004; Van der Schoot et al., 2009). For example, Lewis and Mayer (1987), Pape (2003), Van der Schoot et al. (2009) and Verschaffel et al. (1992) showed that children find it easier to convert the relation term ‘more than’ to a subtraction operation (as in the example above) than the relational term ‘less than’ to an addition operation. This effect has been explained by assuming that the semantic memory representation of ‘less than’ is more complex than that of ‘more than’, an effect which is known as the lexical marking principle (Clark, 1969). The reason behind this effect is that the marked relational term (‘less than’) and unmarked relational term (‘more than’) differ in their frequency of occurrence (\*\*French, 1979; Goodwin & Johnson-Laird, 2005; Schriefers, 1990). Whereas the marked term is used only in its contrastive, ‘negative’ sense (‘Peter has less marbles than David’), the unmarked term is used in two senses: the contrastive, ‘positive’ sense (‘Peter has more marbles than David’) but also a neutral, nominal sense (‘Does she have more than one child?’). For word problem solving, the implication is that the memory representation of ‘less than’ is more ‘fixed’ than the memory representation of ‘more than’ (Van der Schoot et al., 2009).

Presumably, the fixedness of its memory representation hinders the problem solvers' ability to reverse 'less than' in the inconsistent condition (in which it primes the inappropriate arithmetic operation). What is of relevance here is that processing a marked relational term such as 'less than' (or 'times less than') is found to be closely associated with reading comprehension abilities (Van der Schoot et al., 2009). In particular, overcoming its semantic complexity and performing the statement reversal are thought to be comprehension-related skills (Kintsch, 1998; Thevenot, 2010). Thus, in this study, reading comprehension is hypothesized to have an indirect effect on word problem solving performance via its influence on relational processing, that is, the mapping of mathematical terms onto mathematical operations (Lee et al., 2004).

However, previous studies have also demonstrated a direct effect between reading comprehension and word problem solving (Pape, 2004; Vilenius-Tuohimaa, Aunola, & Nurmi, 2008). Presumably, general reading comprehension abilities are important in dealing with the linguistic–semantic word problem characteristics such as the semantic structure of a word problem, the sequence of the known elements in the problem text, and the degree in which the semantic relations between the given and the unknown quantities of the problem are stated explicitly (De Corte et al., 1985). All these word problem characteristics have been shown to have an effect on children's solution processes (e.g., De Corte & Verschaffel, 1987; De Corte et al., 1985; Sjøvik, Frostrad, & Heggberget, 1999).

Given that they are grounded in different processing domains (visual–spatial and linguistic–semantic), the two major component skills in word problem solving (production of visual–schematic representation and relational processing) are hypothesized to be unrelated in this study. Yet, the basic abilities which are presumed to underlie these component skills, respectively spatial ability and reading comprehension, are expected to be connected. This hypothesis is based on studies which indicate that both abilities share some cognitive elements like working memory (Ackerman, Beier, & Boyle, 2005; Hannon & Daneman, 2001; Shah & Miyake, 1996) and general intelligence (Ackerman et al., 2005; Keith, Reynolds, Patel, & Ridley, 2008), as well as on the large body of studies which accentuate the importance of spatial ability in the production of non-linguistic situation models during reading comprehension (Haenggi, Kintsch, & Gernsbacher, 1995; Kendeou, Papadopoulos, & Spanoudis, 2012; Kintsch, 1998; Phillips, Jarrold, Baddeley, Grant, & Karmiloff-Smith, 2004; Plass, Chun, Mayer, & Leutner, 2003). Nonetheless, we do not expect the relation between spatial ability and reading comprehension to bring about a direct relation between the two component skills. This expectation is based on the assumption that the direct relationship between these component skills is weak and will therefore vanish in the presence of (the relationship between) the basic abilities.

### 1.3. The present study

A path model for successful word problem solving is established on the basis of the two component skills and their underlying basic abilities as discussed above. The complete path model is represented in Fig. 1. The upper part of the model involves constructs in the visuo-spatial domain – that is, visual–schematic representations and spatial ability – while the lower part involves constructs in the linguistic–semantic domain, that is, relational processing and reading comprehension. Of note is that within both domains direct and indirect paths are hypothesized. Furthermore, a correlation between both basic abilities is captured in the path model.

While all separate relations in our proposed model have been previously investigated in earlier studies, the present study is unique as it combines the component skills and basic underlying abilities from both processing domains in one model. The results obtained from this study can broaden our knowledge of the factors

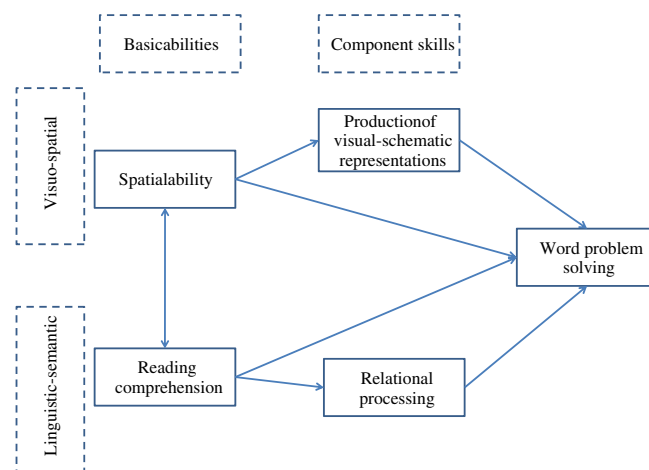


Fig. 1. Path model with all hypothesized pathways.

that are important for word problem solving and can provide an interesting starting point for an effective word problem solving instruction.

The aim of the present study is twofold:

- (1) Investigate whether the component skills and basic abilities in the two processing domains explain unique variance in students' word problem solving skills.
- (2) Examine the direct and indirect (via the component skills) effects of the basic abilities on word problem solving.

## 2. Method

### 2.1. Sample

The study contained data from 128 Dutch sixth grade students (64 boys,  $M_{age} = 11.73$  years,  $SD_{age} = 0.43$  years and 64 girls,  $M_{age} = 11.72$  years,  $SD_{age} = 0.39$  years) from eight elementary schools in The Netherlands. These eight schools were randomly drawn from a total of 20 elementary schools. Approximately 15 students of each of the eight elementary schools were selected on the basis of their proficiency score on the CITO Mathematics test (2008). The CITO Mathematics test is a nationwide standardized test (developed by the Institute for Educational Measurement) to follow students' general math ability during their elementary school career. On the basis of this test the students are equally divided in low, average and high math performers to obtain a representative sample. Parents provided written informed consent based on printed information about the purpose of the study.

### 2.2. Instruments and measurement procedure

The measurement instruments that were used in this study were administrated to the students by three trained independent research-assistants in two sessions of approximately 45 and 30 min.

#### 2.2.1. Word problem solving performance

Word problem solving performance were examined with the Mathematical Processing Instrument (MPI), translated to Dutch. The MPI consisted of 14 word problems based on previous studies (Hegarty & Kozhevnikov, 1999; Van Garderen & Montague, 2003, see Appendix A). The internal consistency coefficient (Cronbach's alpha) of this instrument, measured in American participants, is .78 (Hegarty & Kozhevnikov, 1999). The Cronbach's alpha of the

MPI in this study was .72. The word problems were printed on cards and presented in four different orders. All problems were read out loud to the students to control for differences in decoding skill. To prevent that the execution of the required arithmetic operations would be a determining factor in students' word problem solving, these operations were easy and could be solved by every student. Furthermore, students were allowed to solve each word problem within 3 min and during this time the experimenter did not speak to the student. To be sure that students had enough time to solve the word problems, a pilot study was conducted with five sixth grade students. The results of the pilot study showed that every student was able to solve each of the 14 items of the MPI within the required 3 min. The number of problems solved correctly was used as the dependent variable in the analysis.

### 2.2.2. Component skill in the visuo-spatial domain: Production of visual-(schematic) representations

After the 3 min of problem solving time, a short interview was held about the nature of the (mental) representation evoked by the word problem. The exact procedure of this interview is adapted from the study of Hegarty and Kozhevnikov (1999). We adjusted some questions of this interview procedure to make sure that children were not forced to make a visual representation, but used the strategy they preferred to solve the word problem (see Appendix B for the interview-format).

For each visual representation a score was obtained expressing whether the students had made a visual-schematic or a pictorial representation. These two representation categories are exemplified by the following word problem:

"Problem 1: A balloon first rose 200 meters from the ground, then moved 100 meters to the east, and then dropped 100 meters. It then traveled 50 meters to the east, and finally dropped straight on the ground. How far was the balloon from its original starting point?"

A representation was coded as visual-schematic if students drew a diagram, used gestures showing spatial relations between elements in a problem in explaining their solution strategy, or reported a spatial image. Fig. 2 shows an example of a visual-schematic representation.

A representation was coded as pictorial if the student drew an image of the objects and/or persons referred to in the problem, rather than the relations between them.

Fig. 3 shows an example of a pictorial representation.

In total 612 representations were made by the students. All representations were coded by three independent coders. In the first coding session 32 representations were randomly selected and coded according to the two categories by all coders. The inter-rater reliability of these 32 coded representations was high (Cohen's Kappa ( $\kappa$ ) = .88, Tabachnick & Fidell, 2006). Because the results of this coding session were good, the remaining representations were coded by all coders in the same way. Because we were only interested in the production of visual-schematic representations, the total number of visual-schematic representations made by each student was included in the analysis.

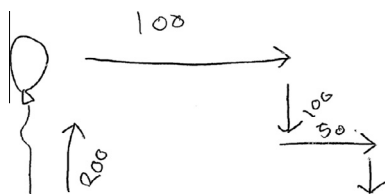


Fig. 2. A visual-schematic representation of word problem 1.



Fig. 3. A pictorial representation of word problem 1.

### 2.2.3. Component skill in the linguistic-semantic domain: Relational processing

To determine relational processing, i.e. the derivation of the correct relations between the solution-relevant elements from the text base of the word problem, we used the inconsistency task. The inconsistency task contained eight two-step compare problems consisting of three sentences, which were selected from the study of Hegarty et al. (1992) and translated into Dutch. The first sentence of each word problem was an assignment statement expressing the value of the first variable, that is, the price of a product at a well-known Dutch store or supermarket (e.g., At Albert Heijn a bottle of olive oil costs 4 euro). The second sentence contained a relational statement expressing the value of the second variable (i.e. the price of this product at another store or supermarket) in relation to the first (e.g., At Spar, a bottle of olive oil costs 3 euro more than at Albert Heijn). In the third sentence, the problem solver was asked to find a multiple of the value of the second variable (e.g., If you need to buy three bottles of olive oil, how much will you pay at Spar?). The answer to these word problems always involved first computing the value of the second variable (e.g.,  $4 + 3 = 7$ ) and then multiplying this solution by the quantity given in the third sentence (e.g.,  $7 \text{ times } 3 = 21$ ). In this task, the consistency of the word problems was manipulated. Consistency refers to whether the relational term in the second sentence was consistent or inconsistent with the required arithmetic operation. A consistent sentence explicitly expressed the value of the second variable (V2) in relation to the first variable (V1) introduced in the prior sentence (At V2, product A costs N euro [more/less] than at V1). An inconsistent sentence related the value of the second variable to the first by using a pronominal reference (This is N euro [more/less] than at V2). Consequently, the relational term in a consistent word problem primed the appropriate arithmetic operation ('more than' when the required operation is addition, and 'less than' when the required operation is subtraction), and the relational term in an inconsistent word problem primed the inappropriate arithmetic operation ('more than' when the required operation is subtraction, and 'less than' when the required operation is addition). We controlled for difficulty in reading comprehension throughout the consistent and inconsistent word problems by balancing the number of unmarked ('more than') and marked ('less than') relational terms. As such, the relatively higher complexity that would have been introduced by an inconsistent item cannot be explained by any effect of markedness.

The numerical values used in the word problems were selected on basis of the following rules in order to control for the difficulty of the required calculations: (1) The answers of the first step of the operation were below 10, (2) The final answers were between the



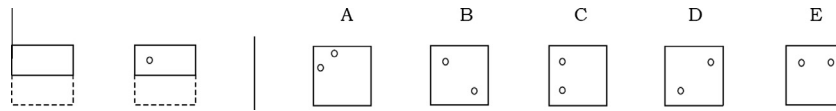


Fig. 4. The Paper Folding task (Ekstrom, French, Harman, & Derman, 1976).

14 and 40, (3) None of the first step or final answers contained a fraction of a number or negative number, (4) No numerical value occurred twice in the same problem, and (5) None of the (possible) answers resulted in 1. The numerical values used in consistent and inconsistent word problems were matched for magnitude.

For the analysis, we looked at the students' accuracy (i.e. the amount of correct answers) on the inconsistent word problems. The internal consistency coefficient of this measure in the present study was high (Cronbach's alpha = .90).

#### 2.2.4. Basic ability in the visuo-spatial domain: Spatial ability

The Paper Folding task (retrieved from The Kit of Factor-Referenced Cognitive Tests; Ekstrom, French, Harman, & Derman, 1976) and the Picture Rotation task (Quaiser-Pohl, 2003) were standardized tasks used to measure spatial visualization. In the Paper Folding task, children were asked to imagine the folding and unfolding of pieces of paper. In each problem in the test, some figures were drawn at the left of a vertical line and there were others drawn at the right. The figures at the left of the vertical line represented a square piece of paper being folded. On the last of these figures one or two small circles were drawn to show where the paper had been punched. Each hole was punched throughout the thicknesses of paper at that point. One of the five figures at the right of the vertical line showed where the holes would be located when the paper was completely unfolded. Children had to decide which one of these figures was correct. This task took 6 min and had a sufficient internal consistency coefficient in the present study (Cronbach's alpha = .70). Fig. 4 shows one of the 20 test items of the Paper Folding task.

In the Picture Rotation task children were asked to rotate a non-manipulated picture of an animal at the left of a vertical line. The three pictures at the right of the vertical line showed the rotated and/or mirrored image of that same animal. One of these three pictures was only rotated; two of these pictures were both rotated and mirrored. Children had to decide which of the three pictures was only rotated. This task took 1.5 min and its internal consistency coefficient in this study was high (Cronbach's alpha = .93). Fig. 5 shows one of the 30 test items of the Picture Rotation task.

To obtain a general measure of spatial ability, the raw scores of each of the spatial ability tasks were rescaled into a z-score. Subsequently, these z-scores were aggregated into an average z-score ( $M = .00$ ,  $SD = .84$ ).

#### 2.2.5. Basic ability in the linguistic-semantic domain: Reading comprehension

The standardized CITO (Institute for Educational Measurement) Reading comprehension test (2010) was used to measure children's reading comprehension skills. Each test contains two modules, each consisting of a text and 25 multiple choice questions. The questions pertained to the word, sentence or text level and tapped both the text base and situational representation that the reader constructed from the text (e.g., Kintsch, 1988). Students' raw test scores on the 50 items were rescaled to a normed proficiency score. The proficiency scores ( $M = 42.06$ ,  $SD = 14.06$ ) made it possible to compare the results of the reading comprehension test with other versions of this test from other years. The internal consistency coefficient of this test in sixth grade students was high

with a Cronbach's alpha of .89 (Weekers, Groenen, Kleintjes, & Feenstra, 2011).

#### 2.3. Data analysis

Path analyses using MPlus Version 4 (Muthén & Muthén, 2006) were performed to examine if the hypothesized model fitted the data. The standard Maximum Likelihood (ML) method of estimating free parameters in structural equation models was used to assess model fit. In this procedure, a non-significant chi-square ( $X^2$ ), a root-mean-square error of approximation (RMSEA) under .05, and a Comparative Fit Index (CFI) value above .95 together indicate a strong fit of the data with the model, while a RMSEA value under .08 and a CFI above .90 indicate an adequate fit (Hu & Bentler, 1999; Kline, 2005). Two path analyses were performed to examine the path model which fitted the data best.

First, the complete hypothesized model (see Fig. 1) was tested, including the two component skills, their underlying basic abilities and their connection with word problem solving performance. This model was considered as the baseline model in the analyses. To examine the presence of mediation by the two component skills, the baseline model, including both direct and indirect effects, was tested against a second model containing only the direct effects (see Fig. 6). If the second model had worse fit indices compared to the baseline model – based on a significant increase of the chi-square statistic (CMIN) –, mediation effects were present (Kline, 2005). The degree in which the effect is reduced is an indicator of the potency of the mediator (Preacher & Hayes, 2008). The value of this indirect effect was calculated with the following formula<sup>1</sup>:

$$B_{\text{indirect}} = B_{(\text{path } a)} * B_{(\text{path } b)}$$

followed by

$$B_{\text{indirect}} / B_{(\text{total})}$$

### 3. Results

#### 3.1. Descriptive statistics

Table 1 presents the means and standard deviations of, and the correlations between, the five measures of this study. This table shows that the correlations between the measures are moderate, except for two correlations. The correlation between the production of visual-schematic representations and students' relational processing skills is negligible ( $r = .08$ ). On the other hand, the correlation between spatial ability and word problem solving is strong ( $r = .59$ ).

<sup>1</sup>  $B_{(\text{path } a)}$ : the unstandardized coefficient from spatial ability/reading comprehension to the production of visual-schematic representations/relational processing.  $B_{(\text{path } b)}$ : the unstandardized coefficient from the production of visual-schematic representations/relational processing to word problem solving performance.  $B_{(\text{total})}$ : direct relation between spatial ability/reading comprehension and word problem solving performance.



Fig. 5. The Picture Rotation task (based on Quaiser-Pohl, 2003).

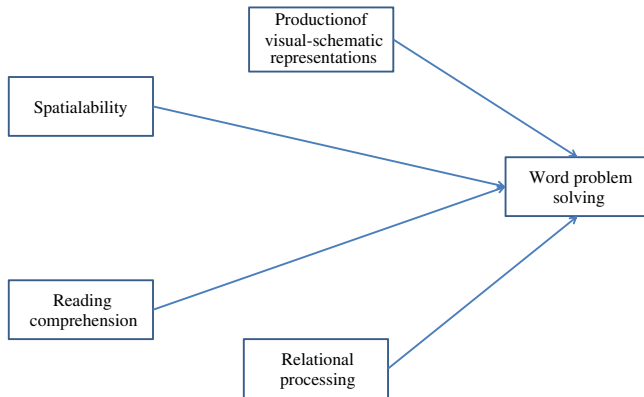


Fig. 6. Model 2, including only the direct effects.

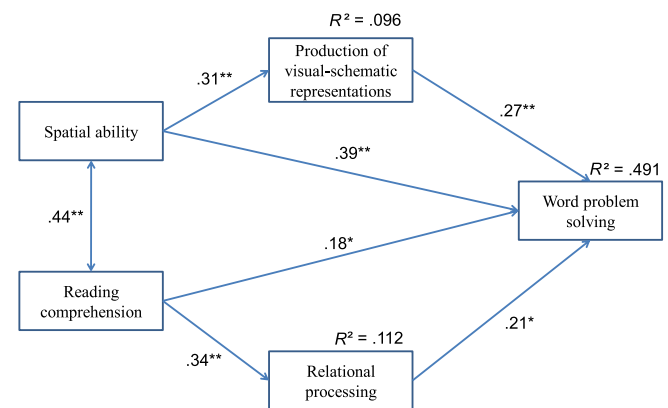


Fig. 7. Hypothesized model, including the standardized estimates of the variables influencing word problem solving performance, the significant pathways are indicated with an asterisk, \* $p < .05$ , \*\* $p < .001$ .

### 3.1.1. Examining the complete hypothesized path model, including direct and indirect effects

The hypothesized path model is assessed with Maximum Likelihood estimation. The fit indices for this baseline model are good:  $X^2(3) = 3.50$ ,  $p = .32$ , CFI = .99 and RMSEA = .04.

Fig. 7 shows the graphical representation of the hypothesized model, including the standardized parameter estimates. Table 2 shows the complete parameter estimates of the model. The path analysis shows that 49.1% ( $R^2 = .491$ ) of the variance in word problem solving performance is explained by the production of visual-schematic representations ( $\beta = .27$ ,  $p < .001$ ), spatial ability ( $\beta = .39$ ,  $p < .001$ ), students' relational processing skills ( $\beta = .21$ ,  $p < .001$ ) and reading comprehension ( $\beta = .18$ ,  $p < .05$ ). This is a large effect size (Tabachnick & Fidell, 2006). Spatial ability ( $\beta = .31$ ,  $p < .001$ ) explains 9.6% ( $R^2 = .096$ ) of the variance in the production of visual-schematic representations and reading comprehension ( $\beta = .34$ ,  $p < .001$ ) explains 11.2% ( $R^2 = .112$ ) of the variance in relational processing. These two effect sizes can be categorized as medium (Tabachnick & Fidell, 2006). Finally, in line with our expectations, the correlation between spatial ability and reading comprehension is significant ( $r = .44$ ,  $p < .001$ ).

### 3.1.2. Testing mediation

In order to test the existence of mediation by the two component skills, the baseline model is tested against a second model, including only the direct effects (see Fig. 6). If the baseline model fits the data better, mediation exists and both direct and indirect effects are present.

Also the second path model is assessed with Maximum Likelihood estimation. This model has a bad model fit:  $X^2(6) = 54.22$ ,  $p < .001$ , CFI = .62, RMSEA = .25. Compared to the baseline model, the second model fits the data less adequately: CMIN (3) = 50.72,  $p < .001$ . This finding indicates that the model with both direct and indirect effects fits the data better than the model with only the direct effects. This means that – at least partial – mediation occurs. Thus, in line with our expectations, spatial ability and reading comprehension have both a direct and indirect relation with word problem solving. The value of the indirect effect of spatial ability can be calculated as follows:

$$B_{\text{indirect}} = B_{(a)} * B_{(b)} = 0.90 \times 0.31 = 0.279, \text{ and}$$

$$B_{\text{indirect}}/B_{\text{total}} = 0.279/1.30 = 0.21.$$

The value of the indirect effect of reading comprehension can be calculated in the same way:

$$B_{\text{indirect}} = B_{(a)} * B_{(b)} = 0.03 \times 0.45 = 0.014, \text{ and}$$

$$B_{\text{indirect}}/B_{\text{total}} = 0.014/0.04 = 0.34.$$

Thus, the production of visual-schematic representations explains 21% of the relation between spatial ability and word problem solving performance. On the other hand, relational processing explains 34% of the relation between reading comprehension and word problem solving performance.

Table 1  
Intercorrelations, means, standard deviations for all measures.

Measure	1.	2.	3.	4.	5.	M	SD
1. Word problem solving performance	–					6.68	2.87
2. Relational processing	.37**	–				2.94	1.27
3. Production of visual-schematic representations	.44**	.08	–			2.13	2.45
4. Reading comprehension	.48**	.33**	.26*	–		42.06	14.06
5. Spatial ability (z-score)	.59**	.24*	.31**	.43**	–	.00	.84

\*  $p < .01$ .

\*\*  $p < .001$ .

**Table 2**

Results from the path analysis, including unstandardized and standardized parameter estimates of the direct pathways.

Pathway		<i>B</i>	<i>SE</i>	$\beta$
Visual-schematic representations	Word problem solving performance	0.31**	.08	.27
Spatial ability	Visual-schematic representations	0.90**	.24	.31
Spatial ability	Word problem solving performance	1.30**	.25	.39
Relational processing	Word problem solving performance	0.45*	.15	.21
Reading comprehension	Relational processing	0.03**	.01	.34
Reading comprehension	Word problem solving performance	0.04*	.02	.18

\*  $p < .05$ .\*\*  $p < .001$ .

#### 4. Discussion

This study examined the importance of two component skills – that is, the production of visual-schematic representations and relational processing – as well as their basic underlying abilities – that is, spatial ability and reading comprehension – for successful word problem solving (e.g., Hegarty & Kozhevnikov, 1999; Van der Schoot et al., 2009; Van Garderen, 2006). The uniqueness of this study lies in the fact that it is the first study that examined these constructs, tapping different processing domains (i.e. visuo-spatial and linguistic-semantic), in one hypothesized path model. Moreover, both direct and indirect effects of spatial ability and reading comprehension were investigated.

In line with previous research, the results of the path analyses showed that the two component skills (i.e. the production of visual-schematic representations and relational processing) explained unique variance in students' word problem solving performance (Hegarty & Kozhevnikov, 1999; Van der Schoot et al., 2009; Van Garderen, 2006). With respect to the direct and indirect effects of the component skills' underlying basic abilities, this study showed that 21% of the relation between spatial ability and word problem solving was explained by the production of visual-schematic representations. Furthermore, 34% of the relation between reading comprehension and word problem solving was explained by relational processing. Overall, the path model explained 49% of the variance in word problem solving.

##### 4.1. Limitations

Two limitations of this study should be mentioned. The first limitation covers the instrument to determine the nature of the visual representations that were made. After each item of the MPI a short interview was held to establish (1) whether a visual representation was made and (2) whether this representation was pictorial or visual-schematic in nature. Although the most visual representations were made on paper during the task ( $M = 3.58$ ), some representations were made mentally ( $M = 1.20$ ). This means that, when the students were asked to describe and draw the pictures they had in their mind while solving the problem (see the interview procedure described in Appendix B), careful observations from the test assistants were essential to disclose these mental representations. Yet, they could not be completely sure if the representation drawn on a piece of paper (asked retrospectively) was an exact copy of the representation that was made in the head of the child during task performance. Videotapes of each test administration were used to facilitate the process of signaling the mental visual representations.

The second limitation pertains to the correlational nature of the data, which makes it impossible to draw conclusions about any causal relationships between basic abilities, component skills and word problem solving performance. The results of this study only show that these variables are associated with each other. Future experimental studies in which the component skills and basic

abilities are manipulated, should make it possible to draw stronger conclusions concerning causal relationships between the processes which are involved in word problem solving.

##### 4.2. Directions for future research

In future research the production of visual-schematic representations and relational processing should be examined in more detail to draw stronger conclusions. For example, we suggest to examine the production and characteristics of visual representations in the light of individual differences, i.e. differences between low, average and high achievers and/or boys and girls. Several authors have found differences between low, average and high achievers in their production of visual representations and word problem skills (e.g., Van Garderen, 2006; Van Garderen & Montague, 2003). In addition, the scientific literature gives indications that boys have better spatial skills than girls (e.g., Casey, Nuttall, Benbow, & Pezaris, 1995; Casey et al., 1997). Therefore, the production of visual-schematic representations might be a more naturally representation strategy for boys compared to girls.

The findings of this study are also interesting for educational practice. Follow-up studies should examine the effects of interventions in which elementary and secondary school students are taught to systematically build visual-schematic (mental) representations during math problem solving. Several studies have shown that it is more effective to teach children to make their own representations, instead of providing representations in advance (e.g., in the form of illustrations, Van Dijk, Van Oers, & Terwel, 2003; Van Dijk, Van Oers, Van den Eeden, & Terwel, 2003). The use of schema-based instruction in word problem solving (e.g., Jitendra, DiPipi, & Perron-Jones, 2002; Jitendra & Hoff, 1996), where students have to map the information onto a relevant schematic diagram after identifying the problem type, might therefore be a less effective manner to increase word problem solving performance. Besides teaching students to produce visual-schematic representations, one should teach students to derive the correct relations between solution-relevant elements from the text base of the word problem. As reading comprehension is found to be essential for this component skill, word problem instruction should not only focus on the strategic aspects of word problem solving, but also on the more linguistic-semantic aspects.

#### Appendix A

The word problems on the Mathematical Processing Instrument (Hegarty and Kozhevnikov, 1999):

1. At each of the two ends of a straight path, a man planted a tree and then every 5 m along the path he planted another tree. The length of the path is 15 m. How many trees were planted?
2. On one side of a scale there is a 1 kg weight and half a brick. On the other side there is one full brick. The scale is balanced. What is the weight of the brick?

3. A balloon first rose 200 m from the ground, then moved 100 m to the east, then dropped 100 m. It then traveled 50 m to the east, and finally dropped straight to the ground. How far was the balloon from its original starting point?
4. In an athletics race, Jim is four meters ahead of Tom and Peter is three meters behind Jim. How far is Peter ahead of Tom?
5. A square (A) has an area of 1 square meter. Another square (B) has sides twice as long. What is the area of B?
6. From a long stick of wood, a man cut 6 short sticks, each 2 feet long. He then found he had a piece of 1 foot long left over. Find the length of the original stick.
7. The area of a rectangular field is 60 square meters. If its length is 10 m, how far would you have traveled if you walked the whole way around the field?
8. Jack, Paul and Brian all have birthdays on the 1st of January, but Jack is 1 year older than Paul and Jack is 3 years younger than Brian. If Brian is 10 years old, how old is Paul?
9. The diameter of a tin of peaches is 10 cm. How many tins will fit in a box 30 cm by 40 cm (one layer only)?
10. Four young trees were set out in a row 10 m apart. A well was situated beside the last tree. A bucket of water is needed to water two trees. How far would a gardener have to walk altogether if he had to water the four trees using only one bucket?
11. A hitchhiker set out on a journey of 60 miles. He walked the first 5 miles and then got a lift from a lorry driver. When the driver dropped him he still had half of his journey to travel. How far had he traveled in the lorry?
12. How many picture frames 6 cm long and 4 cm wide can be made from a piece of framing 200 cm long?
13. On one side of a scale there are three pots of jam and a 100 g weight. On the other side there are a 200 g and a 500 g weight. The scale is balanced. What is the weight of a pot of jam?
14. A ship was North-West. It made a turn of 90° to the right. An hour later it made a turn through 45° to the left. In what direction was it then traveling?

## Appendix B

These interview questions were used by the examiner:  
If a picture was made on paper

1. How did your picture of the problem help you get the answer?

[The child answers the question and moves onto the next word problem]

If no picture was made on paper

1. How did you solve the problem?

If the child describes a mental picture or uses gestures.

1. Describe the picture you had in your mind while you were solving the problem, and make the picture on the paper.
2. How did your picture of the problem help you get the answer?

[The child answers the questions and moves onto the next word problem]

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